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Discrete Levels Density and Spin Cut off Parameters of the ^{36}Cl, ^{40}K, ^{60}Co and ^{61}Ni Nucleuses

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Abstract: On the bases of Bethe and the constant temperature models, the level density parameters have been calculated for the ^{36}Cl, ^{40}K, ^{60}Co and ^{61}Ni nuclei, nucleuses through fitting the complete level schemes in low excitation energy levels. Both calculations reproduce experimental level densities equally well. The spins cut off parameters of the same nuclei have been determined from the analysis of experimental data for spins of low-lying states.

Keywords: ***Nuclear Level Density, Excited Levels, Spin Cut off Parameter, Potassium 40, Chlorine 36, Cobalt 60, Nichel 61***

1- Introduction

The nuclear levels density parameters are the essential tools for investigating nuclear structure. Basically, predicting distribution of the excited levels of nucleus representing formidable challenge of our understanding this complicated quantum system. On the other hand, the level densities are very important ingredients in related with areas of Physics and Technology for instance in astrophysics for determining thermonuclear rates for nucleosynthesis, or in fission/fusion reactor designs. Since Bethe's pioneering work [1], many studies have been devoted to evaluation of the nuclear level density (NLD). The so-called partition function method is by far the most widely used technique for calculating level densities, particularly in view of its ability to provide simple analytical formulae. In its simplest form, the NLD is evaluated for noninteracting gas fermions which confined to the nuclear volume and having equally spaced energy levels. Such a model corresponds to the zeroth-order approximation of Fermi gas model and leads to very simple analytical though unreliable expressions for the NLD. In an attempt to reproduce the experimental data, various phenomenological modifications for original analytical formulation of Bethe have been suggested, in particular to allow for shell and pairing effects.

This led, first, to the constant temperature formula, then to the shifted Fermi gas model and later to the popular back shifted fermi gas model [2, 3].

In the present work the energy levels and spins of extensive level schemes are applied as: (a) to determine the spin cut off parameter for low-lying levels; and (b) to obtain the level density parameters for the constant temperature and Bethe formulae.

2- Statistical Formula

The nuclear temperature (T) can be defined by the nuclear level density ρ(E) [4].

$$\frac{1}{T} = \frac{d}{dE}\ln \rho(E) \qquad (1)$$

Integration yields the constant temperature Fermi gas formula [2, 5].

$$\rho(E) = \frac{1}{T}\exp(\frac{E - E_0}{T}) \qquad (2)$$

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The nuclear temperature (T) and the ground state back shift ($E_\circ$) can be determined by using the experimental data on the level density.

The Bethe formula of the level density [6] for the back-shifted Fermi gas model [7, 8] can be written as:

$$\rho(E) = \frac{e^{2\sqrt{a(E-E_1)}}}{12\sqrt{2}\sigma\, a^{1/4}(E-E_1)^{5/4}} \qquad (3)$$

In this case, the level density parameter (a) and the ground state back shift (E_1) are obtained by fitting the experimental results. The distribution of spins J is determined by the spin cut-off parameter σ^2 [2, 6] as:

$$f(J) = \exp\left(\frac{-J^2}{2\sigma^2}\right) - \exp\left(\frac{-(J+1)^2}{2\sigma^2}\right) \approx \frac{2J+1}{2\sigma^2}\exp\left[-\frac{(J+1/2)^2}{2\sigma^2}\right] \qquad (4)$$

With this spin distribution, the spin-dependent level density is geven by

$$\rho(E,J) = \rho(E) f(J) \qquad (5)$$

σ^2 is related to the effective moment of inertia (I_{eff}) and to nuclear temperature (T) [7, 9] as:

$$\sigma^2 = \frac{I_{eff} T}{\hbar^2} \qquad (6)$$

The nuclear moment of inertia for a rigid body is given by $I_{Rigid} = \frac{2}{5}MR^2$, where M=A is the amu nuclear mass, $R=1.25A^{1/3}$ fm, is the nuclear radius. They resulted in [9],

$$\sigma^2 = 0.0150 A^{5/3} T \qquad (7)$$

Gilbert and Cameron [2] calculated the spin cut-off parameter for the Bethe formula with a reduced moment of inertia,

$$\sigma^2 = 0.0888 A^{2/3}\sqrt{a(E-E_1)} \qquad (8)$$

3- Fit of Level Density Formulae

Each of the two level density formula has two free parameters. They may be obtained by fitting the experimental measured level schemes (Fig. 1). We have applied these formulas on measured new level scheme for the ^{36}Cl, ^{40}K, ^{60}Co and ^{61}Ni nuclei [10]. Our best fit values have been obtained by using Bethe and constant temperature formula. The results are tabulated in Table 1. The accumulated levels, N(E) as a function of energy are plotted in Fig. 1. According to these figures, the agreement between theory and experiment is admissible and both formulae fit the measured level scheme equally well [5, 7, 8].

Furthermore, the spin cut off parameter σ^2 has been obtained by fitting the spin distribution with the theoretical expression (4).

Our best fit values of spin cut off parameters are σ^2=5.38 for ^{36}Cl, σ^2=4.32 for ^{40}K, σ^2=7.30 for ^{60}C and σ^2=6.40 for ^{61}Ni. These deduced values are very different from their corresponding rigid body values of σ^2=8.27 for ^{36}Cl, σ^2=6.91 for ^{40}K, σ^2=10.92 for ^{60}Co and σ^2=11.92 for ^{61}Ni. These fittings are against the claim made by some authors that the spin cut off parameter reduces its rigid body value at lower energy levels.

Table 1. Level density parameters of the Bethe formula and the constant temperature formula

Nuclide	Parameters of Bethe formula		Parameters of CT model	
	a [MeV^{-1}]	E_1 [MeV]	T [MeV]	$E_\circ$ [MeV]
^{36}Cl	3.460	-1.503	1.953	-2.662
^{40}K	4.9	-0.875	1.369	-1.5891
^{60}Co	6.2	-1.789	1.190	-2.6747
^{61}Ni	5.1	-1.345	1.342	-1.9634

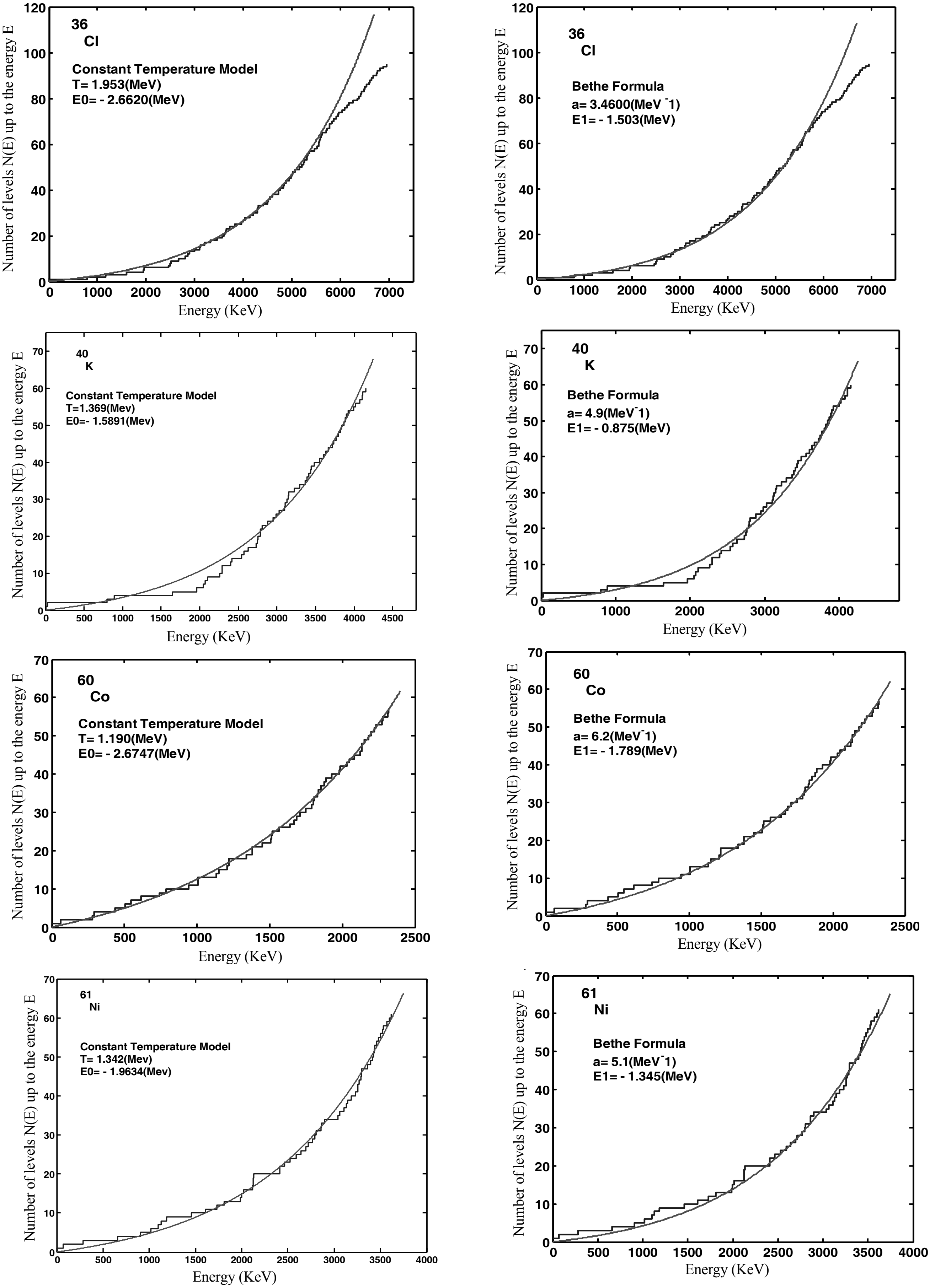

Fig. 1. Plot of the Number of levels N(E) up to the energy E (histograms [10]) for ^{36}Cl, ^{40}K, ^{60}Co and ^{61}Ni together with fitted curves calculated with the Bethe and constant temperature formulae (solid curves).

4- Conclusion

The complete and extensive nuclear level schemes of four nuclei, provide sufficient basis for statistical interpretations of low energy nuclear level schemes and various tests of statistical theories. The level densities near the ground state and the neutron binding energy are well reproduced by the Bethe formula. Also they are as well as the constant temperature formula, if the two free parameters are fitted.

Then, spin cut-off parameters of ^{36}Cl, ^{40}K, ^{60}Co and ^{61}Ni have been determined by analysing the experimental data on spins of low-lying states. They are not confirmed with their corresponding rigid body values. The most determinations of the moments of inertia lead to values between half-rigid and rigid–body values.

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